Book review

Algorithms for worst–case design and applications to risk management, by Berç Rustem and Melendres Howe

(Princeton University Press 2002, Princeton NJ, ISBN 0–691–09154–4)

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This book consists of two main parts: Chapters 1 through 7 present the basic minimax model formulation and develop solution algorithms. This is done by following the best tradition of the British nonlinear programming school. On the one hand, the theory and algorithms are presented on an appropriately high level of rigor for the intended readership. For their proposed methods, the authors provide complete convergence and speed–of–convergence analysis. On the other hand, the authors only recommend algorithms which they have implemented and tested themselves. The methods are presented in a sufficiently detailed form, such that interested readers can readily implement their own version. Due to the careful discussion concerning implementation, the reader has quite good chances to tailor the algorithms to the special structure of her own minimax problems.

The second part, consisting of Chapters 8 through 11, presents selected applications of the minimax concept, to financial risk management. The empirical evidence concerning the good performance of the proposed methods, documented in the first part, is further strengthened by successfully applying them to real–life problems. The models are presented and discussed in a detailed fashion, about one half of the book is devoted to this subject. Thus, interested readers can readily implement the proposed models in their own risk management environment. Beyond that, the details are sufficient for building robust models in other fields of applications, by analogy. An outstanding feature of this part is the way, in which the robust models are being built. Instead of replacing the original, well–established economic objectives by minimax, the authors propose models which add a robust component to existing models. This way the integration of a robust decision support strategy into a risk management environment is greatly enhanced.

Concerning this book, the keyword in the financial risk management context is *model risk*. This is considered as part of the operational risk and can roughly be explained as follows. Financial institutions rely heavily on mathematical models in their risk management procedures. Clearly, any model is an imperfect reflection of reality. Model risk is the risk of loss resulting from using an inadequate mathematical model, or from errors in determining the numerical parameters in the model (called calibration in the financial context). A typical source of the latter type of errors is using an inadequate statistical estimation procedure. The models and algorithms, presented in the book, provide a machinery for dealing with model risk. Although the book appeared in 2002, in the meantime dealing with model risk became an increasingly important constituent of risk management, thus strengthening the need for the models and techniques in the book.

Chapter 1 of the book presents the basic minimax model

$$\min_{x \in \mathcal{R}^n} \max_{y \in \mathcal{Y}} f(x, y)$$

where $x \in \mathcal{R}^n$ is the vector of decision variables and $y \in \mathcal{R}^m$ represents uncertain variables constrained to the bounded feasible set $\mathcal{Y}$. Two cases are distinguished: *continuous minimax* corresponds to the case when $\mathcal{Y}$ is a set of continuous values while *discrete minimax* means that $\mathcal{Y}$ is a finite set. Introducing the notation $\Phi(x) := \max_{y \in \mathcal{Y}} f(x, y)$, the above problem can be written as the nonlinear programming problem

$$\min_{x \in \mathcal{R}^n} \Phi(x).$$

The need for special–purpose algorithms for this problem has its roots in the fact that, even in the case when f is smooth, Φ is generally non–smooth when the maximizer is not unique in the definition of Φ.

The subsequent Chapters 2 through 7 present algorithms for the solution of both kinds of minimax problems, including the special case of saddle–point problems. In this first part of the book, the main contributions of the authors are the method in Chapter 4 for continuous minimax and the algorithms in Chapters 6 and 7 for discrete minimax. The algorithm in Chapter 4 is an adaptation of a quasi–Newton method to minimax problems, for which the authors provide a thorough convergence analysis. In the subsequent chapter comparative computational results are reported with the algorithm, involving 21 difficult test problems, and with quite favorable computational results for the proposed method. Chapters 6 and 7 focus on discrete minimax. A sequential quadratic programming (SQP) method is proposed. Constraints on the decision vector x are allowed, they are accounted for by employing an augmented Lagrangian framework. As in the continuous case, a thorough convergence analysis is provided, including the speed of convergence.

The second part of the book begins with Chapter 8, which discusses several robust extensions for hedging strategies, regarding European call options. One of the implications of the continuous–time Black–Scholes theory of option pricing is that, for the seller of the option, the risk can be perfectly eliminated (called a perfect hedge) in the following way: the seller holds an appropriate amount N of the security which underlies the option, where calculating N is based on the delta hedging formula. For a perfect hedge, the seller of the option has to rebalance continuously. The theory works under the assumption that continuous trading is possible. In practice, however, this is not so and, additionally, rebalancing also implies transaction costs. Consequently, rebalancing is performed at discrete intervals of time, resulting in discrete–time delta hedging. This implies a hedging error; the hedge is no more perfect. In other words, by employing discrete–time delta hedging, the seller of the call option encounters model risk. The authors propose several robust hedging strategies, based on continuous minimax models, for dealing with this type of model risk. An outstanding feature of the proposed approach is that it incorporates also transaction costs. Results of an extensive simulation study are presented, for the purpose of comparing the performance of discrete delta hedging with the robust hedging strategies. In addition, an illustrative hedging problem involving British call options is presented, which serves for testing the performance of the authors' algorithm on real–life problems.

Chapter 9 is devoted to the asset allocation problem. The minimax extension of the classical and widely used mean–variance approach of Markowitz is discussed first. For this approach, estimations for expected asset returns and for the covariance matrix of asset returns are needed. This is where model risk enters the scene; different statistical estimation procedures will typically deliver different values for these parameters. The authors call these rival scenarios, and propose minimax models for handle this situation. The next subject is benchmark tracking, where asset allocation aims at replicating the performance of a benchmark which may be, for instance, an index. The authors extend this approach to tracking two or more benchmarks simultaneously. For dual benchmark tracking (two benchmarks), the objective is to minimize the maximum deviation relative to the two benchmarks, under lower bound constraints for the expected excess returns relative to the benchmarks. Both the robust mean–variance and the benchmark tracking approaches are illustrated by bond portfolio selection problems. Subsequently, the authors briefly discuss robust extensions to some further asset allocation techniques and also consider the robust multi–stage extension of the mean–variance approach.

Chapter 10 discusses asset and liability management (ALM) problems. Besides the risk/reward considerations in asset allocation, ALM problems also involve the management of liabilities. Liabilities are typically also random, cash–flows from assets are used to ensure the payments of liabilities. Thus, in addition to risks associated with asset allocation, a further risk component enters via the liabilities. ALM models consider assets and liabilities simultaneously and have a multi–period structure. The traditional approaches to these problems are cash–flow matching and immunization models, the latter focusing on matching changes in liabilities with changes in asset values. Cash flows being explicitly involved, interest rates and the yield curve play an important role. In the first part of the chapter, the authors propose robust extensions to immunization models, with the scenarios corresponding now to different realizations of the yield curve. The second part discusses stochastic models of the cash flow matching type. First a stochastic cash flow matching model is proposed, involving a crossover to the immunization approach. Finally, the stochastic multistage recourse ALM model of Consigli and Dempster is discussed and a minimax formulation of this model is presented.

The final Chapter 11 contains robust models for currency management. The basic idea is to employ a mean-variance approach for currency portfolios, embedded in a minimax framework. The main source of parameter estimation errors is now the estimation of the exchange rate. The minimax model minimizes the mean–variance objective, and maximization is now with respect to the error term in a long–term model for the exchange rates. Point estimates for the exchange rate are typically inaccurate, therefore, in the proposed approach an interval for the error term is employed. The authors present several model variants and discuss the interplay between strategic benchmarks and tactical currency management.

Summarizing, this is an excellent book, which should be on the bookshelf of anyone with interests in minimax or in risk management. In particular, I highly recommend the book to practitioners in financial risk management, facing model risk in their work.